

Exercise 1 : Binary Independence Model Ranking

The Binary Independence Model (BIM) is a probabilistic retrieval model that assumes binary term occurrences and independence between terms. The relevance ranking function ρ_{BIM} under the BIM can be expressed as

$$\begin{aligned}
 \rho_{\text{BIM}}(\mathbf{d}, \mathbf{q}) &= \mathbb{P}(r = 1 | \mathbf{d}, \mathbf{q}) \\
 &\propto \sum_{t \in \mathbf{q}: t \in \mathbf{d}} \omega_{\text{BIM}}(t, D) \\
 &= \sum_{t \in \mathbf{q}: t \in \mathbf{d}} \log \frac{|D| - |D_t| + 0.5}{|D_t| + 0.5} \quad \text{IR:III-73} \\
 &\stackrel{\text{rank}}{=} \sum_{t \in \mathbf{q}: t \in \mathbf{d}} \log \frac{1 - s_t}{s_t}, \quad \text{IR:III-[48-63]}
 \end{aligned}$$

where $s_t = \frac{|D_t| + 0.5}{|D| + 1}$ (IR:III-64) denotes the relevance score of document d with respect to query q . D_t denotes the set of documents in D that contain term t .

A user issues the query $q = (a, c, h)$. The document collection D consists of the following six documents (term occurrences are indicated by listings in parentheses):

$$D = \{d_1 = (a, b, c, b, d), d_2 = (b, e, f, b), d_3 = (b, g, c, d), d_4 = (b, d, e), d_5 = (a, b, e, g), d_6 = (b, g, h)\}$$

- (a) List some of the assumptions made by the Binary Independence Model.
- (b) What does $\omega_{\text{BIM}}(t, D)$ approximate?
- (c) Complete Table 1 with the missing values based on the document collection D .

Table 1: Term statistics for the Binary Independence Model

Term t	a	b	c	d	e	f	g	h
$ D_t $								
s_t								

s_t values rounded to one decimal place.

- (d) Complete Table 2 by calculating the relevance scores $\rho_{\text{BIM}}(\mathbf{d}_i, \mathbf{q})$ for all documents $d_i \in D$ and ranking (i.e., correctly order them) the documents accordingly.

Table 2: Document ranking according to relevance score $\rho_{\text{BIM}}(\mathbf{d}_i, \mathbf{q})$

Document i	$\rho_{\text{BIM}}(\mathbf{d}_i, \mathbf{q})$	Ranking
d_1		
d_2		
d_3		
d_4		
d_5		
d_6		

$\rho_{\text{BIM}}(\mathbf{d}_i, \mathbf{q})$ values rounded to four decimal places.

- (e) Why is the first ranked document at the first position?

Exercise 2 : Language Model Ranking

We denote the relevance function of language models by $\rho_{\text{LM}}(d, q)$. Recall the *query likelihood model* (IR:III-176) with *Jelinek–Mercer* (JM) and *Dirichlet smoothing* (IR:III-188):

$$\begin{aligned}
 \rho_{\text{LM}}(d, q) &= P(d|q) \stackrel{\text{rank}}{=} P(d) \cdot P(q|d) && \text{IR:III-176} \\
 &\stackrel{\text{rank}}{=} P(d) \cdot \prod_{i=1}^{|q|} P(t_i|d) && \text{IR:III-178} \\
 &\stackrel{\text{rank}}{=} P(d) \cdot \sum_{i=1}^{|q|} \log P(t_i|d) && \text{IR:III-181} \\
 &\stackrel{\text{rank}}{=} P(d) \cdot \sum_{i=1}^{|q|} \log(1 - \lambda) \cdot P(t|d) + \lambda \cdot P(t|D) && \text{JM IR:III-188} \\
 &\stackrel{\text{rank}}{=} P(d) \cdot \sum_{i=1}^{|q|} \log \frac{\text{tf}(t, d) + \alpha \cdot P(t|D)}{|d| + \alpha}, && \text{Dirichlet IR:III-188}
 \end{aligned}$$

where $P(t|D)$ denotes the probability of term t in the entire document collection D , i.e., $\frac{\sum_{d \in D} \text{tf}(t, d)}{\sum_{d \in D} |d|}$ (IR:III-184), and λ and α are smoothing parameters.

The query is $q = \text{information retrieval}$, and the document collection D consists of documents d_1, d_2, \dots with the following term frequencies $\text{tf}(t, d)$:

	information	retrieval	...	$\sum_{t \in T} \text{tf}(t, d_i)$
d_1	15	25	...	$ d_1 = 1800$
d_2	15	1	...	$ d_2 = 2000$
\vdots	\vdots	\vdots	\ddots	\vdots
$\sum_{d \in D} \text{tf}(t, d)$	160 000	2400	...	$\sum_{d \in D} d = 10^9$

For example, $\text{tf}(\text{information}, d_1) = 15$ indicates that the term `information` occurs 15 times in document d_1 .

- Given the following document collection D and query q , compute the *Dirichlet* smoothed query likelihood scores ρ_{LM} for documents d_1 and d_2 and rank them accordingly. Use a smoothing parameter of $\alpha = 2000$.
- What is the difference between *Jelinek–Mercer* smoothing and *Dirichlet* smoothing?
- Which smoothing method is more effective for (1) verbose, (2) keyword queries?