Evaluation

Exercise 1 : Rule-Based Learning in 2D

Consider the problem of rule-based learning in the following, rather different, feature space: The set of possible examples is given by all points of the x-y plane with integer coordinates from the interval $[1, 10]$. The hypothesis space is given by the set of all rectangles. A rectangle is defined by the points (x_1, y_1) and (x_2, y_2) (bottom left and upper right corner). Hypotheses are written as $\theta = (x_1, y_1, x_2, y_2)$, and assign a point (x, y) to the value 1, if $x_1 \leq x \leq x_2$ and $y_1 \leq y \leq y_2$ hold, with arbitrary, but fixed integer values for x_1, y_1, x_2, y_2 from the interval $[1, 10]$.

Hint: The maximally specific hypothesis h_{s_0} corresponds to a "zero-sized" rectangle that doesn't contain any points with integer coordinates; you may use the symbol $(\perp, \perp, \perp, \perp)$.

(a) For the setting described above, formulate the most general hypothesis h_{g_0}.

Answer

$h_{g_0} = (1, 1, 10, 10)$

(b) Clarify for yourself how the "more-general" relation $\geq_g$ works in this setting, and check all that apply:

- [ ] $(1, 2, 3, 4) \geq_g (1, 1, 4, 4)$
- [x] $(2, 3, 6, 7) \geq_g (3, 4, 5, 7)$
- [ ] $(1, 1, 2, 8) \geq_g (1, 1, 3, 3)$
- [ ] $(3, 3, 9, 9) \geq_g (1, 1, 1, 1)$

Answer

The relation $\geq_g$ corresponds to the subset relation for rectangles.

(c) Given a hypothesis $h : \theta = (2, 3, 5, 7)$, and an example $\mathbf{x} = (2, 7)$ with $c = 0$, determine two hypotheses h_1 and h_2 such that both are minimal specializations of h, and both are consistent with $(\mathbf{x}, c)$.

Hint: for the correct answers h_i, there must not exist any hypothesis h' consistent with $(\mathbf{x}, c)$ where $h \geq_g h'$ and $h' \geq_g h_i$.

Answer

$h_1 : \theta = (3, 3, 5, 7)$

$h_2 : \theta = (2, 3, 5, 6)$

(or vice-versa)

Exercise 2 : Precision and Recall

In which of the following classification tasks do we aim for high precision, in which for high recall? Why?

(a) Explosive detection using an airport x-ray machine.

Answer

High recall, because we must detect every explosive, with the cost of some false alarms.

(b) Youtube video recommendations (classifying videos as relevant).

Answer

High precision, because missing a relevant video is no problem, but giving many irrelevant videos is.

(c) Choosing a good seat on a half-full train.

Answer

High precision, because not choosing a certain good seat still leaves you plenty of options, but you really don't want to spend the next hours at a bad seat.

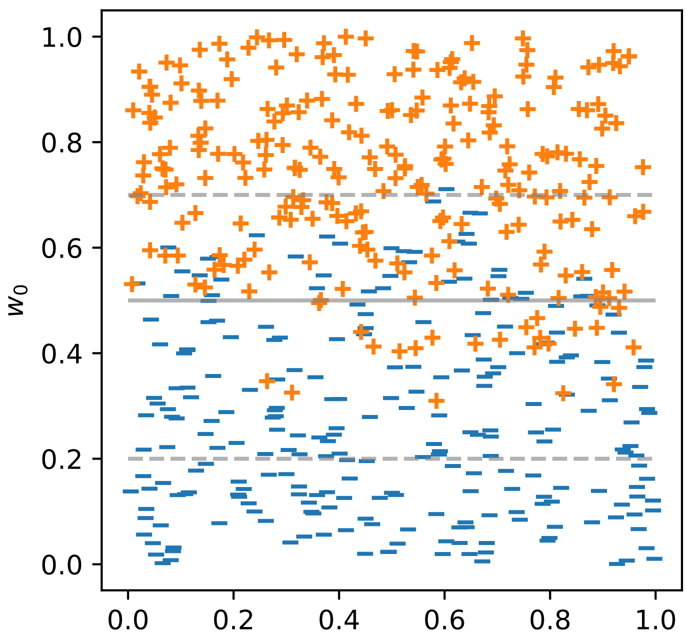
(d) Spell checking (spelling error detection).

Answer

High recall, because ignoring false alarms has lower costs than missing a typo.

Exercise 3 : ROC Curve

Consider the following binary classification scenario:



We use different linear classifiers (horizontal lines) that are parameterized by w_0. Consider the effect of the choice of w_0 on the following two performance metrics:

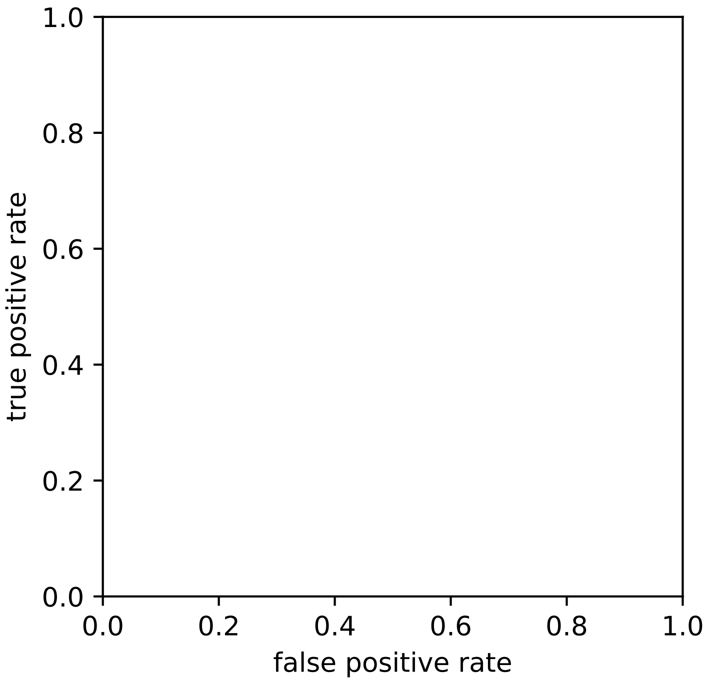
The false positive rate, defined as

$$FPR = \frac{FP}{FP+TN} = \frac{FP}{N} = \frac{|\{(\mathbf{x},c) \in D : y(\mathbf{x}) = 1 \land c = 0\}|}{|\{(\mathbf{x},c) \in D : c = 0\}|},$$

and the true positive rate (i.e. recall), defined as

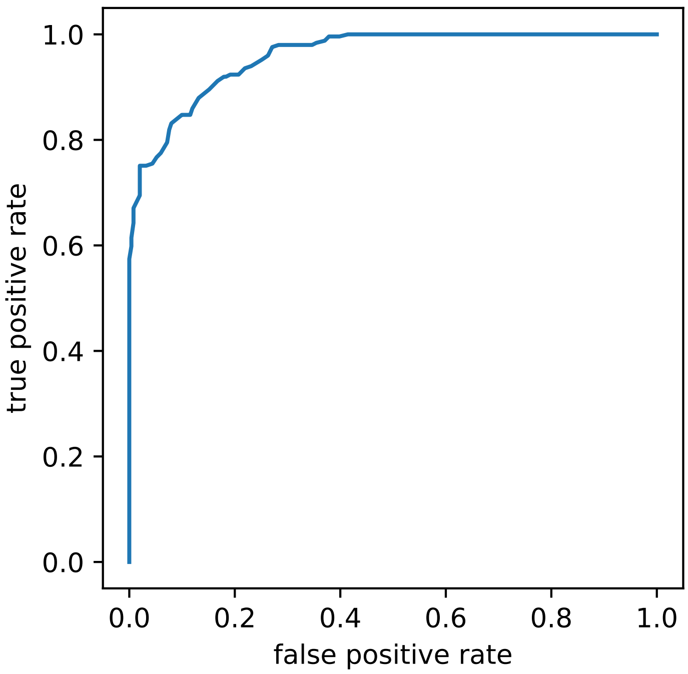
$$TPR = \frac{TP}{TP+FN} = \frac{TP}{P} = \frac{|\{(\mathbf{x},c) \in D : y(\mathbf{x}) = 1 \land c = 1\}|}{|\{(\mathbf{x},c) \in D : c = 1\}|}.$$

(a) Vary w_0 and fill out the following plot:



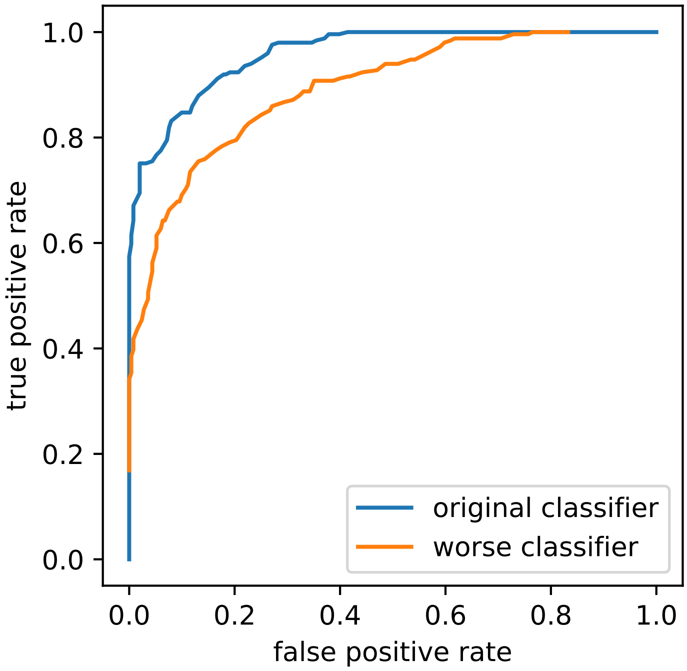
This is called ROC curve (receiver operating characteristic).

Answer



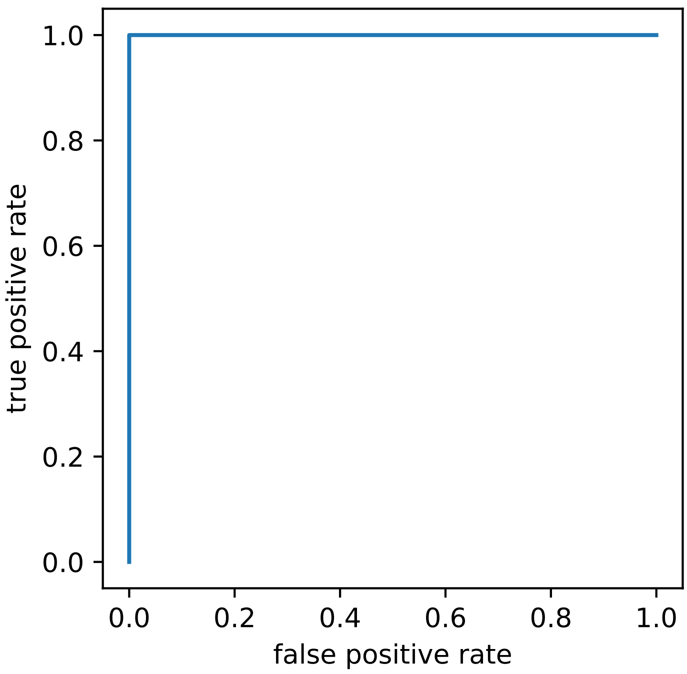
(b) How would the ROC curve of a slightly worse classifier (e.g., one that is not horizontal) look like?

Answer



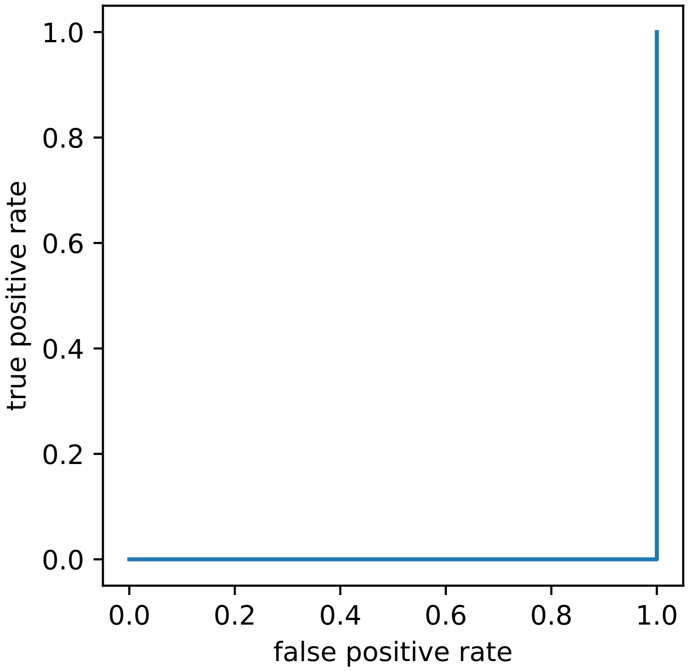
(c) How does the ROC curve of the optimal classifier look like?

Answer



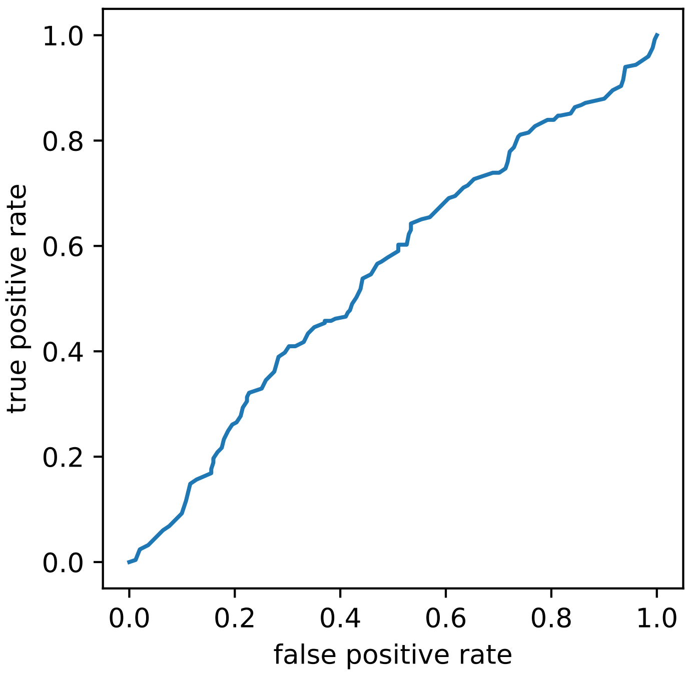
(d) How does the ROC curve of the worst possible classifier look like?

Answer



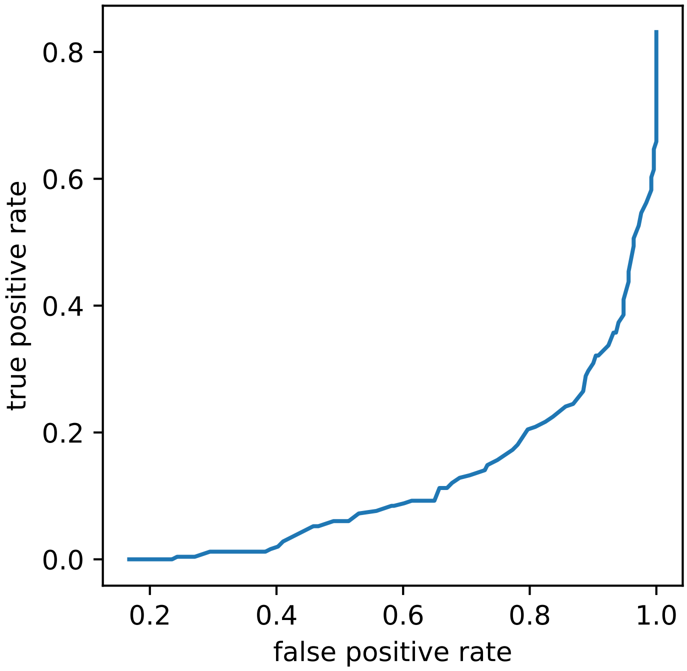
(e) What does the ROC curve of a random classifier look like that uses the threshold parameter as its acceptance probability?

Answer



(f) Imagine a classifier with a ROC curve worse than random guessing. What went wrong here? How could this error be fixed?

Answer



This means that the classes were swapped during training. By inverting the model output, one is able to obtain a model better than random.

(g) How does this relate to forming a classifier from a regression model? Use the terms of bias and threshold.

Answer

By choosing a threshold when forming a classifier from a regression model, one is able to model preferences regarding FPR and TPR, but also regarding precision and recall. This introduces bias.