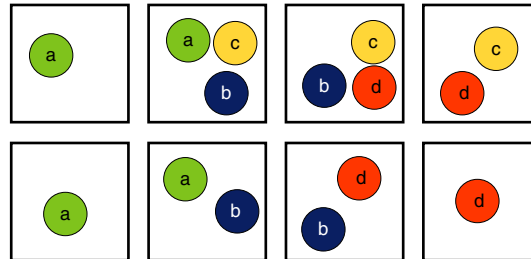


Exercise 1 : Probability Basics (Conditional Independence)

There are eight boxes containing different colored balls as shown in the illustration below:



The balls can be green, blue, yellow, or red (also marked a, b, c, d in the figure). When picking one of the eight boxes at random, let  $A$  refer to the event “box contains a green ball,”  $B$  to the event “box contains a blue ball,”  $C$  to the event “box contains a yellow ball,” and  $D$  to the event “box contains a red ball.” Hence,  $A \cap B$  is the event “box contains both a green and a blue ball,” etc.

- (a) Calculate  $P(A)$ ,  $P(B)$ ,  $P(C)$ , and  $P(D)$ .
- (b) Calculate  $P(A \cap B)$ ,  $P(A \cap C)$ ,  $P(B \cap C)$ , and  $P(B \cap D)$ .
- (c) Check all that apply:
  - The events  $A$  and  $B$  are statistically independent.
  - The events  $A$  and  $C$  are statistically independent.
  - The events  $B$  and  $C$  are statistically independent.
  - The events  $B$  and  $D$  are statistically independent.
- (d) Calculate  $P(A | C)$ ,  $P(B | C)$ , and  $P(A \cap B | C)$ .
- (e) Calculate  $P(B | D)$ ,  $P(C | D)$ , and  $P(B \cap C | D)$ .
- (f) Check all that apply:
  - The events  $A$  and  $B$  are conditionally independent given  $C$ .
  - The events  $B$  and  $C$  are conditionally independent given  $D$ .

Exercise 2 : Bayes' Rule

A hospital database contains diagnoses (diseases) along with observed symptoms:

Patient	Diagnosis	Symptoms								
		$S_1$	$S_2$	$S_3$	$S_4$	$S_5$	$S_6$	$S_7$	$S_8$	$S_9$
1	$C_1$	✓		✓		✓				
2	$C_2$		✓		✓	✓		✓		
3	$C_3$	✓		✓			✓		✓	
4	$C_4$		✓		✓	✓		✓		
5	$C_3$	✓		✓					✓	
6	$C_5$					✓				✓
7	$C_3$	✓		✓			✓			
8	$C_2$		✓					✓		

- (a) Compute the prior probabilities  $P(C_i)$ .
- (b) Compute the posterior probabilities  $P(C_i | S_4)$  of the diagnoses  $C_i$  given symptom  $S_4$ .

Exercise 3 : Naïve Bayes

Given is the following dataset to classify whether a dog is dangerous or well-behaved in character:

Color	Fur	Size	Character ( $C$ )
brown	ragged	small	well-behaved
black	ragged	big	dangerous
black	smooth	big	dangerous
black	curly	small	well-behaved
white	curly	small	well-behaved
white	smooth	small	dangerous
red	ragged	big	well-behaved

- (a) Determine the parameters for a Naïve Bayes classifier on this dataset.
- (b) Classify the new example (Color=black, Fur=ragged, Size=small) using your Naïve Bayes classifier.