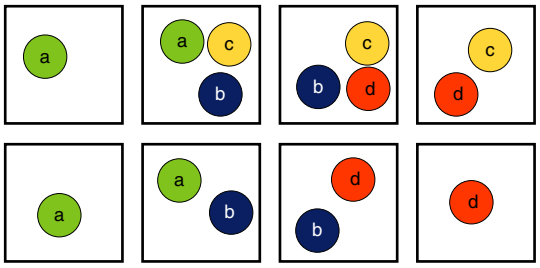
Bayesian Classification

Exercise 1 : Probability Basics (Conditional Independence)

There are eight boxes containing different colored balls as shown in the illustration below:



The balls can be green, blue, yellow, or red (also marked a, b, c, d in the figure). When picking one of the eight boxes at random, let A refer to the event "box contains a green ball," B to the event "box contains a blue ball," C to the event "box contains a yellow ball," and D to the event "box contains a red ball." Hence, $A \cap B$ is the event "box contains both a green and a blue ball," etc.

(a) Calculate $P(A)$, $P(B)$, $P(C)$, and $P(D)$.

(b) Calculate $P(A \cap B)$, $P(A \cap C)$, $P(B \cap C)$, and $P(B \cap D)$.

(c) Check all that apply:

- ☐ The events A and B are statistically independent.
- ☐ The events A and C are statistically independent.
- ☐ The events B and C are statistically independent.
- ☐ The events B and D are statistically independent.

(d) Calculate $P(A \mid C)$, $P(B \mid C)$, and $P(A \cap B \mid C)$.

(e) Calculate $P(B \mid D)$, $P(C \mid D)$, and $P(B \cap C \mid D)$

(f) Check all that apply:

- ☐ The events A and B are conditionally independent given C.
- ☐ The events B and C are conditionally independent given D.

Exercise 2 : Bayes' Rule

A hospital database contains diagnoses ($C_1 \ldots C_5$) for 8 patients along with binary observations of symptoms $S_1 \ldots S_9$:

Patient	Diagnosis	Symptoms								
		S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
1	C_1	1	0	1	0	1	0	0	0	0
2	C_2	0	1	0	1	1	0	1	0	0
3	C_3	1	0	1	0	0	1	0	1	0
4	C_4	0	1	0	1	1	0	1	0	0
5	C_3	1	0	1	0	0	0	0	1	0
6	C_5	0	0	0	0	1	0	0	0	1
7	C_3	1	0	1	0	0	1	0	0	0
8	C_2	0	1	0	0	0	0	1	0	0

(a) Compute based on the database the prior probabilities $P(C_i)$ for each diagnosis.

(b) Compute based on the database the posterior probabilities $P(C_i \mid S_4)$ for each diagnosis.

Exercise 3 : Naïve Bayes

Given is the following dataset to classify whether a dog is dangerous or well-behaved in character:

Color	Fur	Size	Character (C)
brown	ragged	small	well-behaved
black	ragged	big	dangerous
black	smooth	big	dangerous
black	curly	small	well-behaved
white	curly	small	well-behaved
white	smooth	small	dangerous
red	ragged	big	well-behaved

(a) Determine the parameters $P(A_i)$ and $P(B_{j=x_j} \mid A_i)$ for a Naïve Bayes classifier on this dataset.

(b) Classify the new example $\mathbf{x} = (\mathsf{black}, \mathsf{ragged}, \mathsf{small})$ using your Naïve Bayes classifier.