

Decision Trees

Exercise 1 : Decision Trees

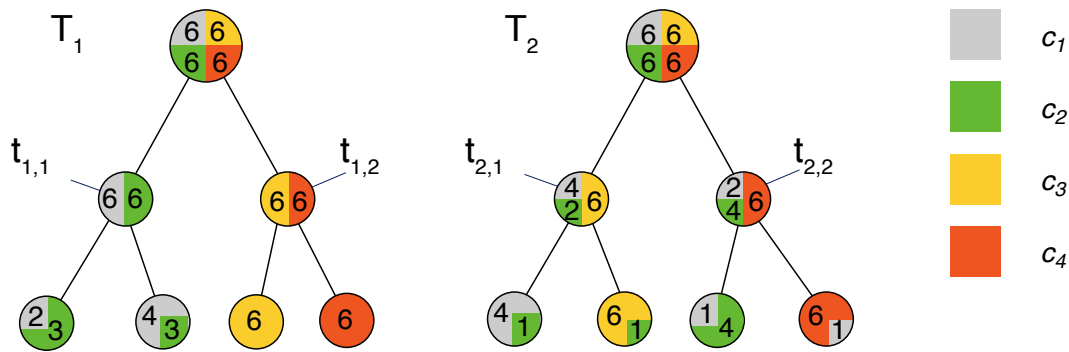
Construct by hand decision trees corresponding to each of the following Boolean formulas. The examples $(\mathbf{x}, c) \in D$ consist of a feature vector \mathbf{x} where each component corresponds to one of the Boolean variables (A, B, \dots) used in the formula, and each example corresponds to one interpretation (i.e. assignment of 0/1 to the Boolean variables). The target concept c is the truth value of the formula given that interpretation. Assume the set D contains examples with all possible combinations of attribute values.

Hint: It may be helpful to write out the set D for each formula as a truth table.

- (a) $A \wedge \neg B$
- (b) $A \text{ XOR } B$
- (c) $A \vee (B \wedge C)$
- (d) $(A \wedge B) \vee (C \wedge D)$

Exercise 2 : Impurity Functions

Let D be a set of examples over a feature space \mathbf{X} and a set of classes $C = \{c_1, c_2, c_3, c_4\}$, with $|D| = 24$. Consider the following illustration of two possible decision trees, T_1 and T_2 – the colors represent the classes present in each subset $D(t_i)$ represented by node $t_{i,j}$ of T_i ; the numbers denote how many examples of each class are present.



- (a) First, consider only the first split that each of the two trees makes: compute $\Delta\iota(D, \{D(t_{1,1}), D(t_{1,2})\})$ and $\Delta\iota(D, \{D(t_{2,1}), D(t_{2,2})\})$ with (1) the misclassification rate $\iota_{misclass}$ and (2) the entropy criterion $\iota_{entropy}$ as splitting criterion. Interpret the results: which of $\{D(t_{1,1}), D(t_{1,2})\}$ or $\{D(t_{2,1}), D(t_{2,2})\}$ is the better first split?
- (b) If we compare T_1 and T_2 in terms of their misclassification rate on D , which one is the better decision tree?
- (c) Assuming the splits shown are the only possibilities, which of T_1 or T_2 would the ID3 algorithm construct, and why?

Exercise 3 : Decision Trees

Given is the following dataset to classify whether a dog is dangerous or well-behaved in character:

| Color | Fur | Size | Character (C) |
|-------|--------|-------|---------------|
| brown | ragged | small | well-behaved |
| black | ragged | big | dangerous |
| black | smooth | big | dangerous |
| black | curly | small | well-behaved |
| white | curly | small | well-behaved |
| white | smooth | small | dangerous |
| red | ragged | big | well-behaved |

- Use the ID3 algorithm with $h_{entropy}$ as the impurity function to determine the tree T .
- Classify the new example (Color=black, Fur=ragged, Size=small) using T .