Neural Networks

Exercise 1 : Gradient Descent

(a) What are the differences between the perceptron training rule and the gradient descent method?

Answer

- The perceptron training rule updates the perceptron weights based on the error in the thresholded perceptron output, which can be 0, 1, or -1. Hence, only the sign of the error is used. Gradient descent updates the weights based on the delta rule, proportional to the error gradient.
- The perceptron training rule converges after a finite number of iterations, provided the training examples are linearly separable. The gradient descent converges asymptotically regardless of whether the training data is linearly separable.

(b) What are the requirements for gradient descent being successful as a learning algorithm?

Answer

- The hypotheses in the hypothesis space H are based on continuous parameters.
- The learning error can be differentiated with respect to hypothesis parameters.
- Ideally, the error function is convex, i.e. it has a single global minimum that gradient descent can converge to (if this is not given, gradient descent can still be used to find a locally optimal solution, but is not guaranteed to find a global optimum)

Exercise 2 : Perceptron Learning

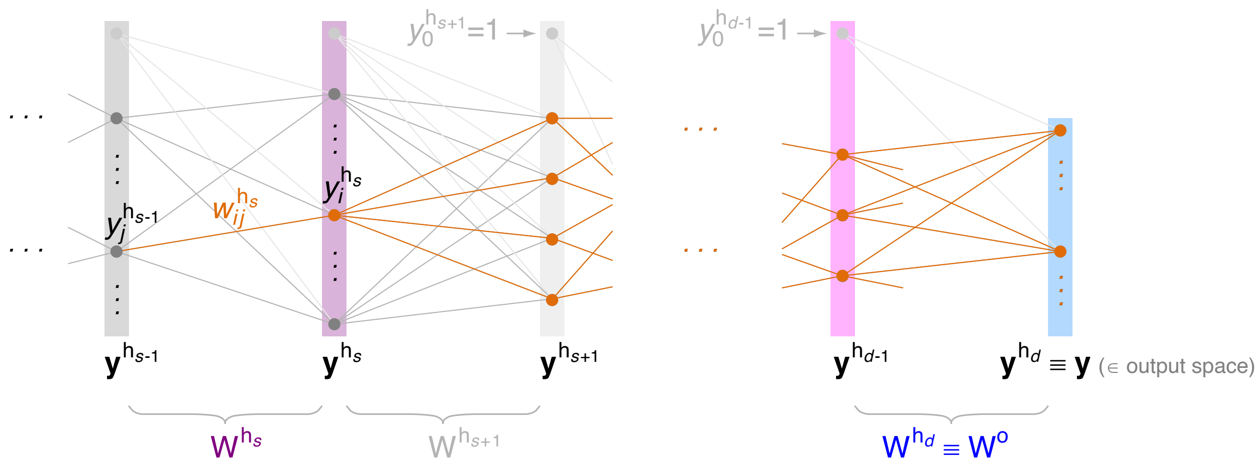
How can perceptrons be applied to solve a classification problem with more than two classes?

Answer

Instead of a thresholded class value, output one value for each class ("logits"). To process these values with a loss, make these outputs probabilities: Map them to the interval $(0, 1)$ and normalize their sum ("softmax"). For the training, use a loss that not only considers the correctness of the most probably class but the whole distribution ("cross-entropy loss"). For the classification step (testing/inference): Find the argmax of the logits.

Exercise 3 : From Chain Rule to Backpropagation

Consider the weight $w_{ij}^{h_s}$ in the following graph of a neural network:



In order to update the weight, we want to compute the derivative $\frac{\partial L(\mathbf{w})}{\partial w_{ij}^{h_s}}$.

- Verify the correctness of the following chain rule:

$$\frac{\partial L}{\partial w_{ij}^{h_s}} = \frac{\partial L}{\partial y_i^{h_s}} \cdot \frac{\partial y_i^{h_s}}{\partial net_i^{h_s}} \cdot \frac{\partial net_i^{h_s}}{\partial w_{ij}^{h_s}} \tag{1}$$

  with $net_i^{h_s} := \sum_j w_{ij}^{h_s} \cdot y_j^{h_s-1}$ and $y_i^{h_s} = \sigma(net_i^{h_s})$.

- What is $\frac{\partial y_i^{h_s}}{\partial net_i^{h_s}}$?

  Answer

  $\frac{\partial y_i^{h_s}}{\partial net_i^{h_s}} = \frac{\partial \sigma(net_i^{h_s})}{\partial net_i^{h_s}}$, i.e., the derivative of the activation function at $net_i^{h_s}$.

- What is $\frac{\partial net_i^{h_s}}{\partial w_{ij}^{h_s}}$?

  Answer

  $\frac{\partial net_i^{h_s}}{\partial w_{ij}^{h_s}} = y_j^{h_s-1}$

- $\frac{\partial L}{\partial y_i^{h_s}}$ can not be computed directly, but is based on results from a previous step. Identify terms from the network this value depends on.

  Answer

  Because all values $\mathbf{y}^{h_{s+1}}$ (within L) depend on the value of $y_i^{h_s}$, we can apply the chain rule for multivariable functions:

$$\begin{aligned} \frac{\partial L}{\partial y_i^{h_s}} &= \sum_l \frac{\partial L}{\partial y_l^{h_{s+1}}} \cdot \frac{\partial y_l^{h_{s+1}}}{\partial net_l^{h_{s+1}}} \cdot \frac{\partial net_l^{h_{s+1}}}{\partial y_i^{h_s}} \\ &= \sum_l \frac{\partial L}{\partial y_l^{h_{s+1}}} \cdot \frac{\partial y_l^{h_{s+1}}}{\partial net_l^{h_{s+1}}} \cdot w_{li}^{h_{s+1}}, \end{aligned}$$

  and these values are equivalent to the terms in Eq. 1, but in a previous step ("to the right" of the current layer in the network, highlighted in orange in the graph above).

- Which of those terms are computed in the forward propagation, which are computed in the backpropagation?

  Answer

  $\frac{\partial y_i^{hs}}{\partial net_i^{hs}}$ and $\frac{\partial net_i^{hs}}{\partial w_{ij}^{hs}}$ can easily be computed in the forward propagation ("from left to right"). Afterwards, the remaining terms can be computed in the backpropagation ("from right to left").